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THE FLOW OF A VISCOUS RAREFIED GAS AROUND A FLAT

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[Following is a translation of an article by A. I. Bunimovich in Izvestia Akademii Nauk SSSR Otdeleniye Tekhnicheskikh Nauk, Mekhanika i Mashinostroyenie, No. 5, 1959, pages 16-18 (Russian, by AIB).]

The problem of a viscous rarefied gas flow about a flat semiinfinite plate is being solved. It is assumed that the viscosity factor at a temperature T is determined by the formula

$$\frac{\mu}{\mu_\infty} = \left(\frac{T}{T_\infty}\right)^n \qquad (0.1)$$

It is shown that the accounting of the sedate dependence of μ from T leads to the deductions of the gas rarefaction influence on the aerodynamic characteristics, qualitatively different from those obtained in the works in which the linear law of μ from T dependence was being investigated (for example V. P. Shidlovskiy (1), and M.F. Shirokova (2).

* * *

1. Let us transform the variables, as proposed by A.A. Dorodnitsin, in the boundary layer equations corresponding to the flow about a plate with a Prandtl number equalling the unit.

$$\xi = x, \qquad \eta = \int_0^y \frac{\rho}{\rho_\infty}\, dy \qquad (1.1)$$

and, in accordance with (1), let us pass to dimensionless variables

$$\lambda = \eta\left(\frac{\nu_\infty \xi}{U}\right)^{-\frac{1}{2}}, \quad \vartheta = \frac{1}{\infty}\left(\frac{\nu_\infty \xi}{U}\right)^{-\frac{1}{2}} \qquad (1.2)$$

and to the sought dimensionless functions

$$\bar{v}_x = \frac{v_x}{U}, \quad \bar{v}_y = \frac{V_y}{U} = \frac{1}{U}\left(\frac{T_\infty}{T} v_y + v_x \frac{\partial \eta}{\partial y}\right), \quad \bar{T} = \frac{T}{T_\infty} \qquad (1.3)$$

(In the following, the stroke above dimensionless variables will be omitted). Here, we shall have, as usual - density, - viscosity factor, v_x, v_y - composing velocities, T - temperature, l - length of the average molecules free run path, U - the creeping flow's velocity, the index ∞ refers to magnitudes corresponding to the unperturbed flow.

By introducing the current ψ function in formulas: (1)

$$v_x = l_\infty \frac{\partial \psi}{\partial \eta} = \frac{\partial \psi}{\partial \lambda} \vartheta, \qquad v_y = -\frac{Ul_\infty^2}{\nu_{\infty}} \frac{\partial \psi}{\partial \xi} = \frac{\vartheta^2}{2}\left(\lambda \frac{\partial \psi}{\partial \lambda} + \frac{\partial \psi}{\partial \vartheta}\right) \quad (1.4)$$

the boundary layer equations will take the form

$$\frac{\partial}{\partial \lambda}\left[T^{n-1}\frac{\partial^2 \psi}{\partial \lambda^2}\right] + \frac{\vartheta^2}{2}\left[\left(\frac{\partial \psi}{\partial \lambda}\right)^2 + \frac{\partial^2 \psi}{\partial \lambda \partial \vartheta}\frac{\partial \psi}{\partial \lambda} - \frac{\partial^2 \psi}{\partial \lambda^2}\frac{\partial \psi}{\partial \vartheta}\right] = 0 \quad (1.5)$$

$$\frac{\partial}{\partial \lambda}\left[T^{n-1}\frac{\partial T}{\partial \lambda}\right] + \frac{\vartheta^2}{2}\left[\frac{\partial T}{\partial \vartheta}\frac{\partial \psi}{\partial \lambda} - \frac{\partial T}{\partial \lambda}\frac{\partial \psi}{\partial \vartheta}\right] + \frac{U^2}{Jc_pT_\infty}\vartheta^2 T^{n-1}\left(\frac{\partial^2 \psi}{\partial \lambda^2}\right)^2 = 0$$

Taking into account the slip and the temperature jumps, they may be written in the form: (4)

$$v_x = rl\frac{\partial v_x}{\partial y}, \quad T = T_n + hrl\frac{\partial T}{\partial y} \quad \text{при} \quad y = 0 \quad (1.6)$$

$$\left(r = 0.998\frac{2-\alpha}{\alpha}, \quad h = \frac{(2/s) - \alpha}{2-\alpha}\frac{2\gamma}{\gamma+1}\right)$$

where α and S - accommodation coefficients, T_w - plate's temperature.

If we take account of the local length of the free run's path with the temperature, the conditions (1.6) may be written

$$\frac{\partial \psi}{\partial \lambda} = r\vartheta T^{n-1/2}\frac{\partial^2 \psi}{\partial \lambda^2}, \quad \psi = \frac{\partial \psi}{\partial \vartheta} = 0, \quad T = T_w + hrT^{n-1/2}\frac{\partial T}{\partial \lambda} \quad \text{при} \quad \lambda = 0 \quad (1.7)$$

2. For the establishment of the solution, and according to (1) let us present the sought functions in the form of ϑ power series.

$$\psi = \varphi_0(\lambda)\vartheta^{-1} + \varphi_1(\lambda) + \ldots \qquad T = T_0(\lambda) + T_1(\lambda)\vartheta + \ldots \quad (2.1)$$

Let us note, that since equations (1.5) and the boundary conditions (1.7) of the examined problem are exact to the ϑ power series approximation, we should limit ourselves to the same ϑ power series approximation in the decompositions (2.1).

Let us substitute the series (2.1) into the equations and the boundary conditions, and assemble along the members of the same ϑ order. We shall obtain:

a) for the zero approximation

$$2(T_0^{n-1}\varphi_0'')' + \varphi_0\varphi_0'' = 0 \quad (2.2)$$

$$2(T_0^{n-1}T_0')' + \varphi_0 T_0' + (\gamma - 1)M_\infty^2 \varphi_0''^2 T_0^{n-1} = 0$$

$$\varphi_0(0) = \varphi_0'(0) = 0, \quad T_0(0) = T_w, \quad \varphi_0'(\infty) = 1, \quad T_0(\infty) = 1$$

b) For the first approximation

$$2(T_o^{n-1}\varphi_1'')' + (\varphi_o\varphi_1')' + 2(n-1)\,(T_o^{n-2}T_1\varphi_o'')' = 0 \qquad (2.3)$$

$$2(T_o^{n-1}T_1)'' + (\varphi_o T_1)' + 2(\gamma-1)\,M_\infty^2\,[(n-1)T_o^{n-2}\varphi_o''^2 T_1 + 2\varphi_o''\varphi_1''T_o^{n-1}] = 0$$

$$\varphi_1(0) = 0,\ \varphi_1'(0) = r\varphi_o''(0)T_w^{n-\frac{1}{2}},\ \varphi_1'(\infty) = 0,\ T_1(\infty) = 0$$

The zero approximation system of equations (2.2) determines the solution of a known boundary layer at flat plate problem, with a constant T_w temperature, and with a flow about it by a compressible gas.

For T_o there is the known (3) formula

$$T_o(\lambda) = T_w - \frac{\gamma-1}{2} M_\infty^2\varphi_o'^2(\lambda) + \left(1 - T_w + \frac{\gamma-1}{2} M_\infty^2\right)\varphi_o'(\lambda) \qquad (2.4)$$

For φo, the equation is solved numerically (compare to example in (3)).

The solution of the first approximation equations determines the corrections resulting from the gas rarefaction. The analysis of those equations shows that if we pose in the boundary conditions (2.4) $h = 1$, the solution may be found in the form

$$\varphi_1(\lambda) = rT_w^{n-\frac{1}{2}}\varphi_o'(\lambda); \qquad T_1(\lambda) = rT_w^{n-\frac{1}{2}}T_o'(\lambda) \qquad (2.5)$$

#3. Taking into account formulas (1.2), 2.4) and (2.5) we shall obtain

$$T(\lambda) = T_w - \frac{\gamma-1}{2} M_\infty^2\varphi_o'^2(\lambda) + \left(1 + \frac{\gamma-1}{2} M_\infty^2 - T_w\right)\varphi_o'(\lambda) +$$
$$+ rT_w^{n-\frac{1}{2}}\left[\left(1 + \frac{\gamma-1}{2} M_\infty^2 - T_w\right)\varphi_o''(\lambda) - (\gamma-1)\varphi_o'(\lambda)\varphi_o''(\lambda)\right]\vartheta \qquad (3.1)$$

This relationship with an up to ϑ^2 order magnitudes coincides with the unknown integral for the conventional boundary layer:

$$T = T_w - \frac{1}{2}(\gamma-1)\,M_\infty^2 V_x^2 + \left(1 + \frac{\gamma-1}{2} M_\infty^2 - T_w\right) V_x \qquad (3.2)$$

As to the slip flow velocity near the plate's surface, we obtain from formulas (1.7), (2.1), (2.5), passing to measurable values

$$(V_x)_{y=o} = Url_\infty\left(\frac{T_w}{T_\infty}\right)^{n-\frac{1}{2}}\left(\frac{U}{V_\infty x}\right)^{\frac{1}{2}}\varphi_o''(0) \qquad (3.3)$$

The friction stress over the plate is determined by the formula

$$\tau_w = U\mu_w \frac{\rho_w}{\rho_\infty} \left(\frac{\nu_\infty x}{U}\right)^{-\frac{1}{2}} \left[\varphi_0''(0) + rl_\infty\left(\frac{\nu_\infty x}{U}\right)^{-\frac{1}{2}} \left(\frac{T_w}{T_\infty}\right)^{n-\frac{1}{2}} \varphi_0''(0)\right] \quad (3.4)$$

or, by substituting the value $\varphi_0'''(0)$ from the first equation (2.2), and utilizing the boundary conditions $\varphi_0(0) = \varphi_0'(0) = 0$; $T_0(0) = T_w$,

$$\tau_w = \mu_\infty \left(\frac{T_w}{T_\infty}\right)^{n-\frac{1}{2}} U\left(\frac{U}{\nu_\infty x}\right)^{\frac{1}{2}} \varphi_0''(0) \times$$

$$\times \left[1 + (1-n)\, rl_\infty\left(\frac{U}{\nu_\infty x}\right)^{\frac{1}{2}} \left(\frac{T_w}{T_\infty}\right)^{n-3/2} \left(\frac{T_{oo}}{T_\infty} - \frac{T_w}{T_\infty}\right) \varphi_0''(0)\right] \quad (3.5)$$

Consequently, the deduction that the slip flow and the temperature jump, at the ν^2 order members approximation, have no influence on the magnitude of the friction stress, which was obtained in numerous works - for example in (1) -, is accurate only on the condition that a law of linear dependence from T is chosen, i.e. n = 1. It also results from formula (3.5). that the gas rarefaction has no influence on friction stress, if the plate's temperature is equal to that of the creeping flow deceleration ($T_{oo} = T_w$).

The relative increase of the resistance factor because of the gas rarefaction influence is determined by the formula

$$\frac{C_{x-} - C_{x+}}{C_{x-}} = 2(1-n)\, r\frac{l_\infty}{L} \left(\frac{T_w}{T_\infty}\right)^{n-3/2} \left(\frac{T_{oo}}{T_\infty} - \frac{T_w}{T_\infty}\right) R^{\frac{1}{2}} \varphi_0''(0) \quad (3.6)$$

where L - the plate's length, $R = UL/\nu_\infty$ - the Reynolds number, and the plus and minus indices are respectively related to the rarefied and to the dense gas

The gas temperature near the plate's surface T_+, according to (3.1)

$$\frac{T_+}{T_\infty} = \frac{T_w}{T_\infty} + rl_\infty \left(\frac{U}{\nu_\infty x}\right)^{\frac{1}{2}} \left(\frac{T_w}{T_\infty}\right)^{n-\frac{1}{2}} \left(\frac{T_{oo}}{T_\infty} - \frac{T_w}{T_\infty}\right) \varphi_0''(0) \quad (3.7)$$

Consequently, if $T_w > T_{oo}$, the rarefaction's influence leads to the decrease of the gas temperature at the wall T_+; if $T_w < T_{oo}$, the gas temperature at the wall increases. When $T_w = T_{oo}$ then $T_+ = T_w$.

The relative heat transfer coefficient is determined by the formula

$$\frac{C_{q-} - C_{q+}}{C_{q+}} = \frac{rl_\infty\, T_w^{\,n-\frac{1}{2}}}{T_{oo} - T_w} \left(\frac{U}{\nu_\infty x}\right)^{\frac{1}{2}} \varphi_0''(0) \times$$

$$\times \left[(1-n)\left(\frac{T_{oo}}{T_\infty} - \frac{T_w}{T_\infty}\right)^2 \frac{T_\infty}{T_w} - (\gamma - 1) M_\infty^{\,2}\right] \quad (3.8)$$

The calculations carried out in the case $n = 0.76$, $\gamma = 1.4$, show that in the range

$$2 \leq M_\infty \leq 10; \; 2 \leq T_w / T_\infty \leq 10,$$

the gas rarefaction leads to the increase of the difference $C_{q-} - C_{q+}$, if $T_w > T_{oo}$, and to a decrease when $T_w < T_{oo}$.

Let us note that the equations examined in the works (1 and 2), are obtained from the system (1.5) if we pose $n = 1$. At the same time, the magnitude $\varphi_o''(0)$ becomes equal to the known Blasius constant $\alpha = 0.3321$.

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